ITERATIVE MODEL IMPROVEMENT FOR MODEL-BASED CONTROL

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Overview

We combine ideas from Identification for Control and Experiment Design tools aiming to maximize the life-time performance of a closed-loop system.

"A model-based controller is progressively improved using system identification. Excitation is given to the system when it is convenient."

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Identification for Control

System running in closed loop, but the control performance is not optimal.

"Improve the control performance while limiting the excitation cost."

An identification experiment followed by the "normal operation"

- Control performance $\mathcal{V}$ depends on the parameter covariance P.
- The parameter covariance P depends on the excitation signal r.
- Excitation cost $\mathcal{E}$ depends on excitation signal r.

A trade-off between the excitation cost $\mathcal{E}$ and the control performance $\mathcal{V}$.

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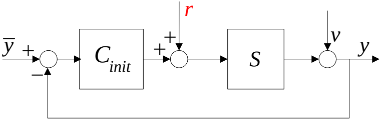

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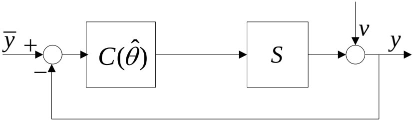

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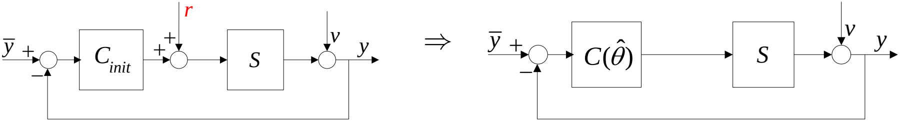

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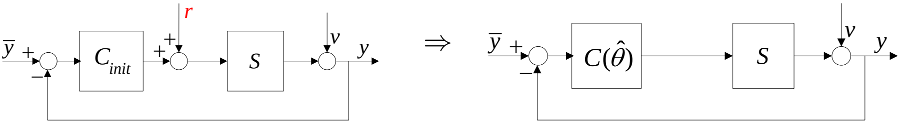

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Experiment Design

For LTI systems

- The covariance P is a nonlinear, nonconvex function of the excitation signal (time domain).
- The information matrix $F = P^{-1}$ is a linear function of the excitation power spectrum $\Phi(\omega)$ (frequency domain).

Input design in the frequency domain using a two-step procedure:

1. Determine an optimal spectrum $\Phi(\omega)$ (convex optimization).
2. Find a signal $r(t)$ with spectrum $\Phi(\omega)$ (stochastic realization).

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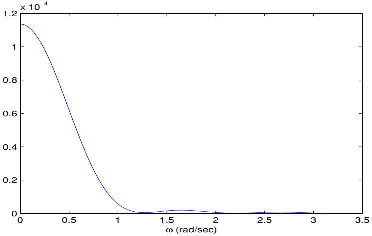

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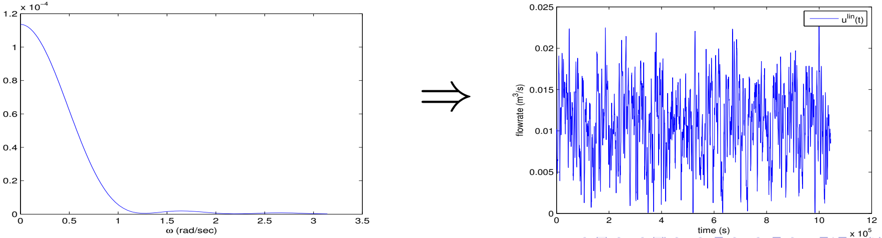

Itendification for Control

Classical: "Given a maximum allowed perturbation, find the excitation signal that gives the best control performance."

$$\max \mathcal{V}(P) \qquad \text{such that} \qquad \mathcal{E} \leq \bar{\mathcal{E}}.$$

M. Gevers and L.Ljung.
Optimal experiment designs with respect to the intended model application.
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Least costly: "Given a minimum allowed performance level, find the excitation signal that minimizes the perturbation."

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Limitations:

- Two distinct phases: identification and normal operation.
- $\mathcal{V}$ and $\mathcal{E}$ considered separately.

"Can we design experiments in such a way that the overall performance is optimized during the whole time of operation?"

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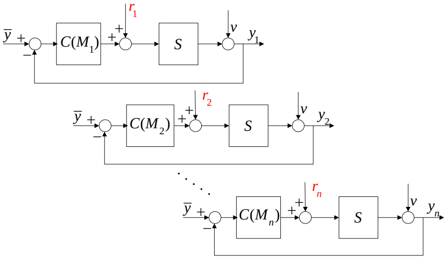
The Framework

Linear system operated in closed-loop over n consecutive learning intervals.

- After an interval, model update and controller re-design.
- Excitation signal r_k in each interval.



Excitation r_k has a dual effect. Worsens performance during the interval k, but can improve performance for interval $k+1$.

"Design the signals r_k to optimize the performance over n intervals."

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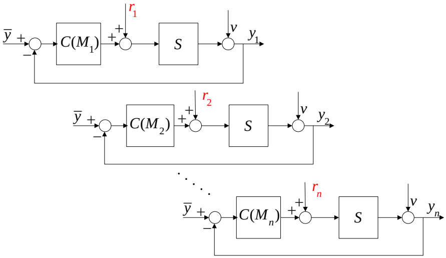

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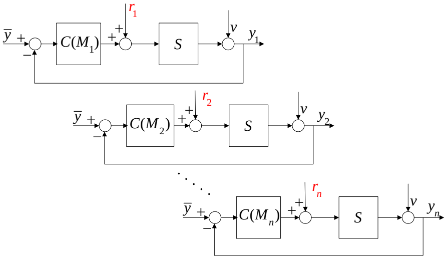

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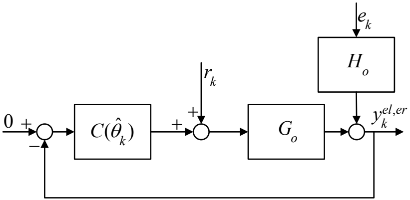
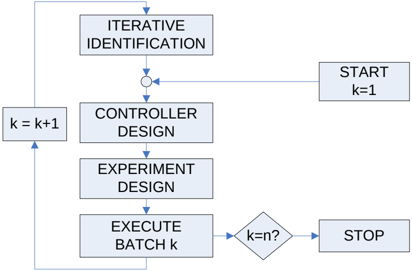


For each learning interval:

- Identification
- Controller design
- Experiment design
- Execute interval k



Analogy with actively adaptive learning control algorithm.

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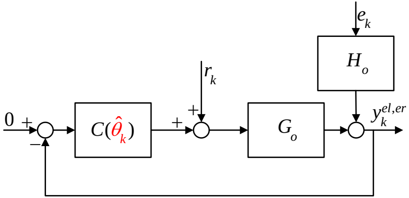

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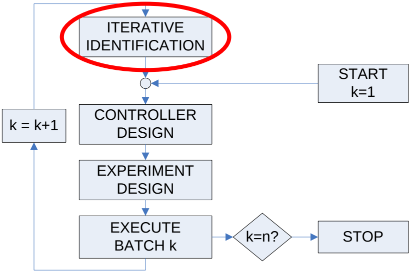

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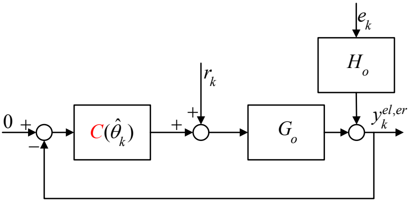

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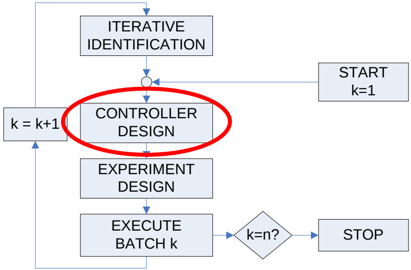

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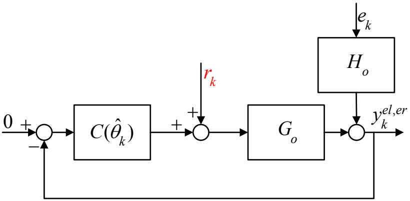

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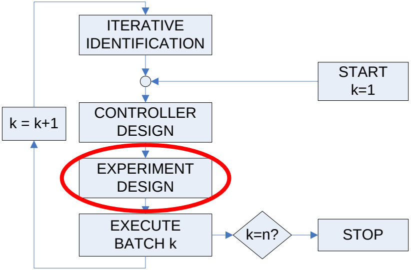

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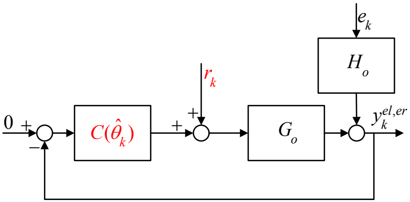

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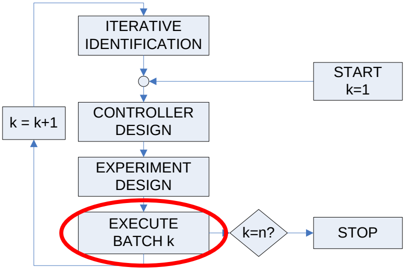

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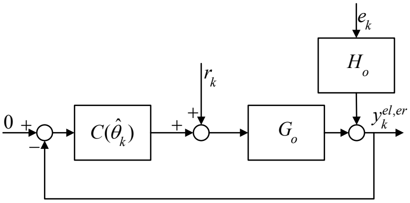

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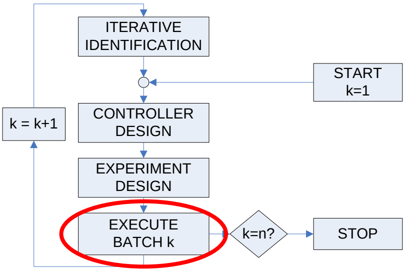

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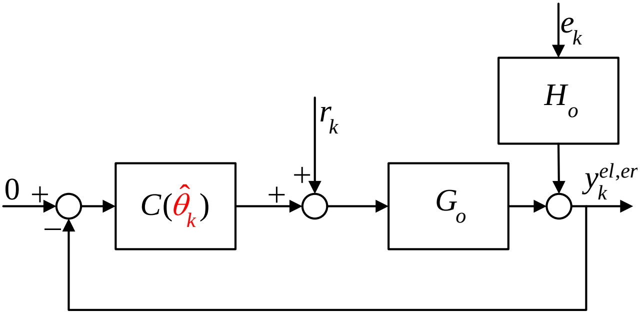

Iterative Identification

After the interval k is executed

- Data (Y_k, U_k) are collected.
- Previous estimate $\hat{\theta}_k \sim \mathcal{N}(\theta_o, P_k)$ is available.

The updated parameter estimate $\hat{\theta}_{k+1}$ is computed as

$$\hat{\theta}_{k+1} = \arg\min_{\theta} \frac{1}{\sigma_e^2} \left\| Y_k - \hat{Y}(U_k, \theta) \right\|_2^2 + \left\| \theta - \hat{\theta}_k \right\|_{P_k^{-1}}^2 .$$

$\theta_o - \hat{\theta}_{k+1} \sim \mathcal{N}(0, P_{k+1}^{-1})$ with $P_{k+1}^{-1} = F_k(\Phi_k) + P_k^{-1}$.

- Information matrix $F_k(\Phi_k)$ linear in the spectrum Φ_k (learning interval sufficiently large).
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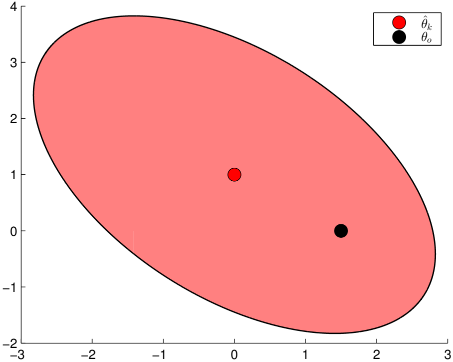
Iterative Identification

We can define an uncertainty region

$$\mathcal{D}_k(\alpha, P_k^{-1}) = \{\theta \in \mathbb{R}^p \mid (\theta - \hat{\theta}_k)^\top P_k^{-1}(\theta - \hat{\theta}_k) \leq \chi_p^2(\alpha)\}$$

where θ_o lies with probability α.



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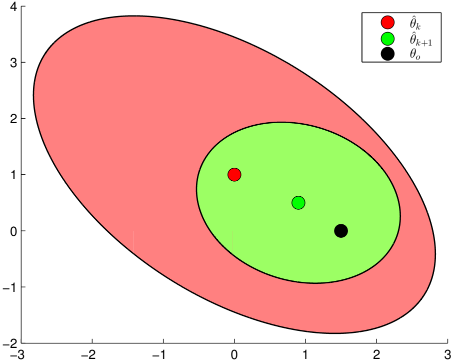

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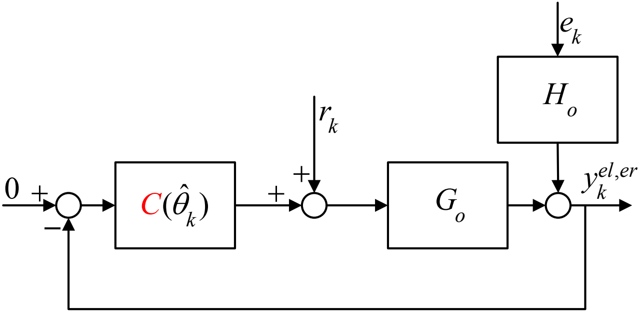

Controller Design

The controller is here designed based on a nominal criterion

$$C_k = C(\hat{\theta}_k)$$

$\mathcal{H}_2$, $\mathcal{H}_\infty$, PID tuning rule,...

Robust stability

The controller will be applied on the true system S_o.

1. Stability of the uncertain controller system can be verified (in the uncertainty set $\mathcal{D}_k$) using established tools.
   - X. Bombois and M. Gevers and L.Ljung.
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     Automatica, 37(10):1651-1636, 2001
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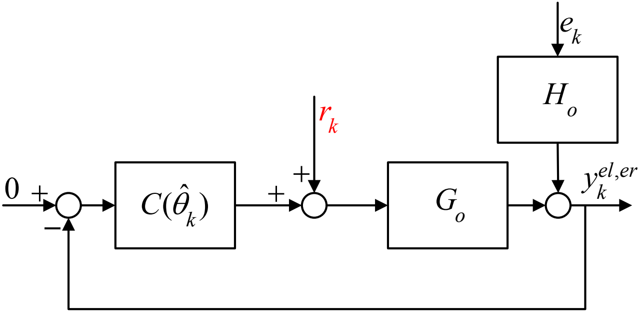

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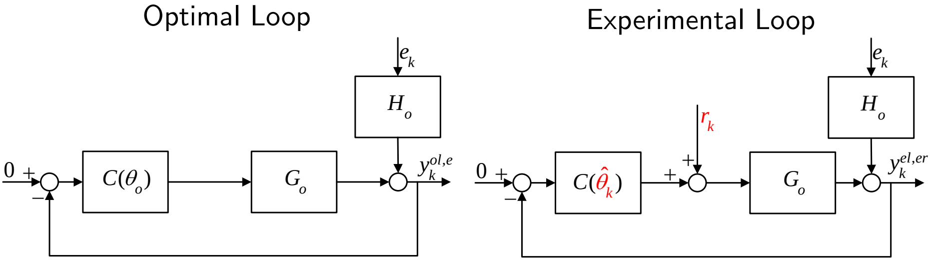
Overview

Let us define:

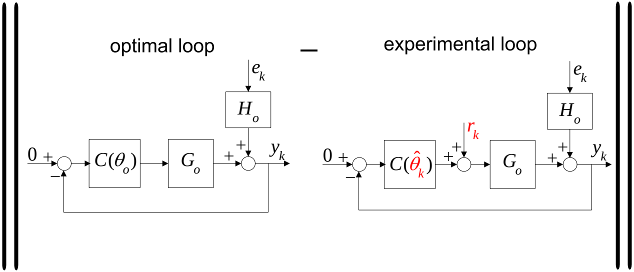


Experiment Design

Objective

Define the total cost for a batch as

$\mathcal{T}_k \triangleq$



Excitation signals r_k are designed in order to

- minimize $\sum_{k=1}^{n} \mathcal{T}_k$.
- satisfy constraints $\mathcal{T}_k \leq \bar{\mathcal{T}}_k$ for each interval.

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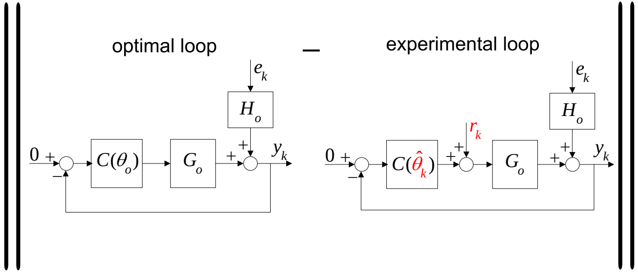

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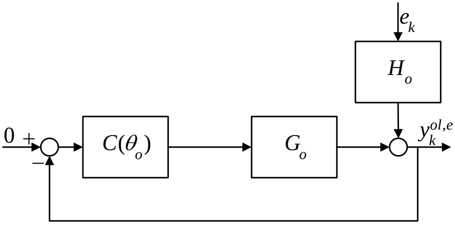
Experiment Design

Total Cost, Modeling Error Cost & Excitation Cost

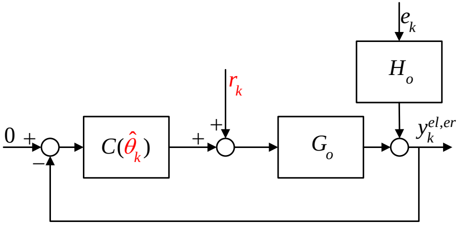
Total Cost: power of output difference between the two loops:

$$\mathcal{T}_k \triangleq E[(y_k^{ol,e} - y_k^{el,er})^2].$$

Optimal Loop



Experimental Loop



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Experiment Design

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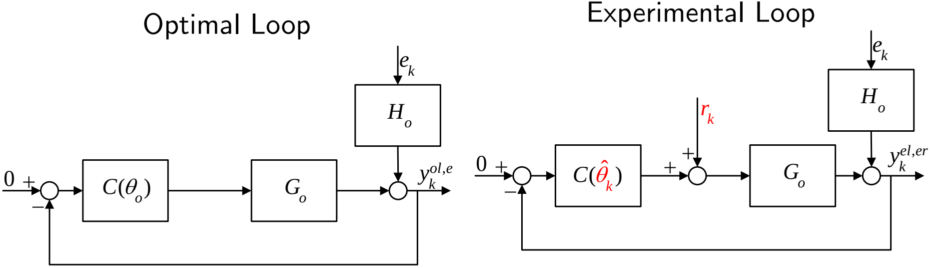

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$$\underbrace{E[(y_k^{ol,e} - y_k^{el,er})^2]}_{\text{Total Cost } \mathcal{T}_k} = \underbrace{E[(y_k^{ol,e} - y_k^{el,e})^2]}_{\text{Modeling Error cost } \mathcal{V}_k} + \underbrace{E[(y_k^{el,r})^2]}_{\text{Excitation Cost } \mathcal{E}_k} .$$

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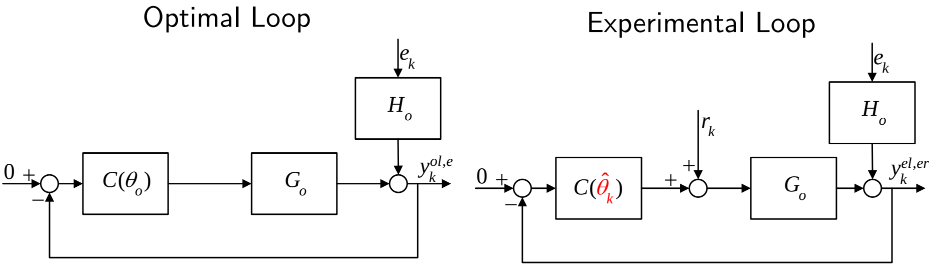

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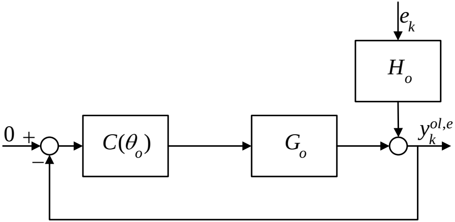

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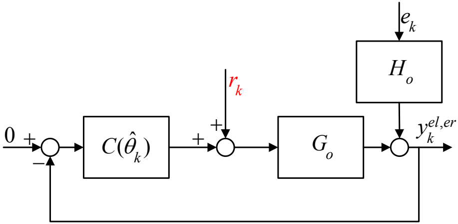

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Experiment Design

Objective

- Experiment Design Problem (for the learning interval 1):
  minimize the summation of the total cost over the future n intervals

$$\min \sum_{k=1}^{n} \mathcal{T}_k \qquad \text{subject to}$$
$$\mathcal{T}_k \leq \bar{\mathcal{T}}_k,\ k = 1, 2, \ldots, n.$$

- Optimization variables: spectra of all the excitation signals $r_1, \ldots, r_n$.
- $\mathcal{T}_k = \mathcal{V}_k + \mathcal{E}_k$ random variables $\Rightarrow$ minimization in a worst-case sense.

  $\mathcal{V}_k^{\mathrm{wc}}$ and $\mathcal{E}_k^{\mathrm{wc}}$ are computed by taking the maximum of their second order approximation over the uncertainty set $\mathcal{D}_k$. $\mathcal{T}_k^{\mathrm{wc}} = \mathcal{V}_k^{\mathrm{wc}} + \mathcal{E}_k^{\mathrm{wc}}$.

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Receding Horizon Implementation

- We use the uncertainty set $\mathcal{D}_k$ to compute $\mathcal{V}_k^{\mathrm{wc}}, \mathcal{E}_k^{\mathrm{wc}}$.
- The uncertainty set $\mathcal{D}_k$ depends on the covariance P_k, which is linear in the spectrum.
- However, the covariance P_k is also a function of θ_o (unknown!). Typical chicken & the egg issue.

Typical solution: replace θ_o with $\hat{\theta}_1$.

Dividing the time in learning intervals allows us to mitigate the effect of this approximation.

The Experiment Design is implemented in Receding Horizon over the learning intervals.

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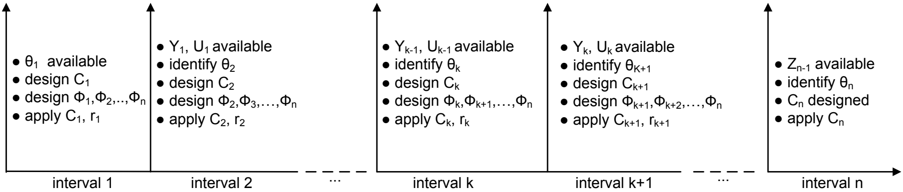

1. ED(1) for interval 1 based on $\hat{\theta}_1$. Spectra $(\Phi_1, \ldots, \Phi_n)$ found. r_1 applied in interval 1. Interval 1 executed, data (Y_1, U_1) collected.
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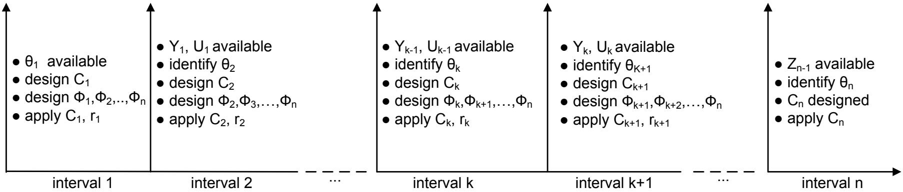
3. ...

Experiment Design

Receding Horizon Implementation



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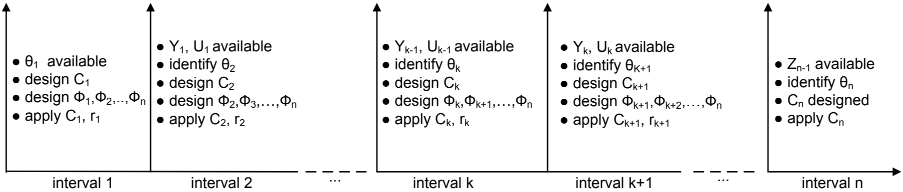

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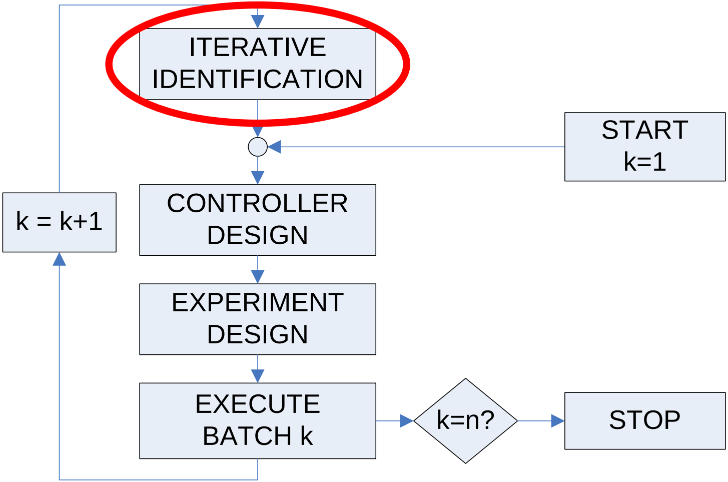
Experiment Design

Flow Diagram



Experiment Design

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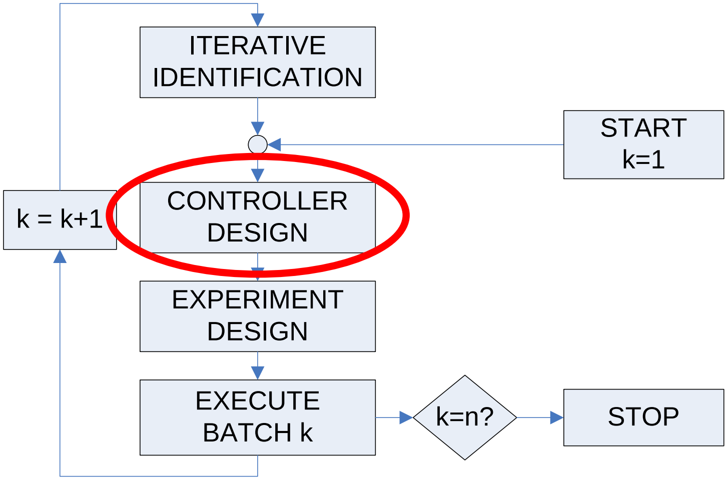

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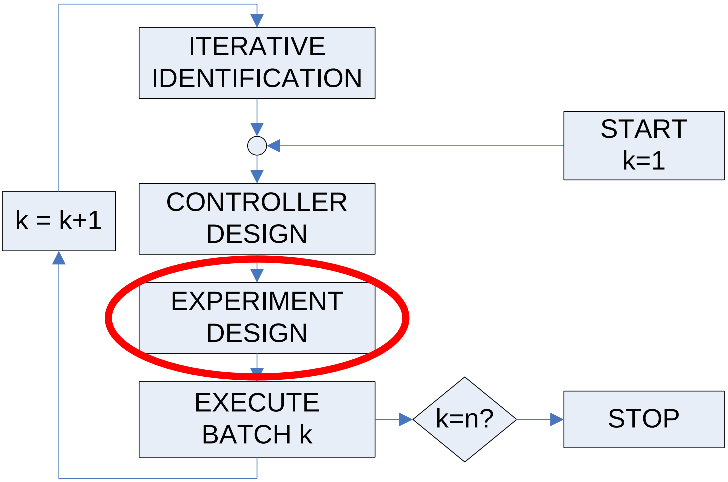

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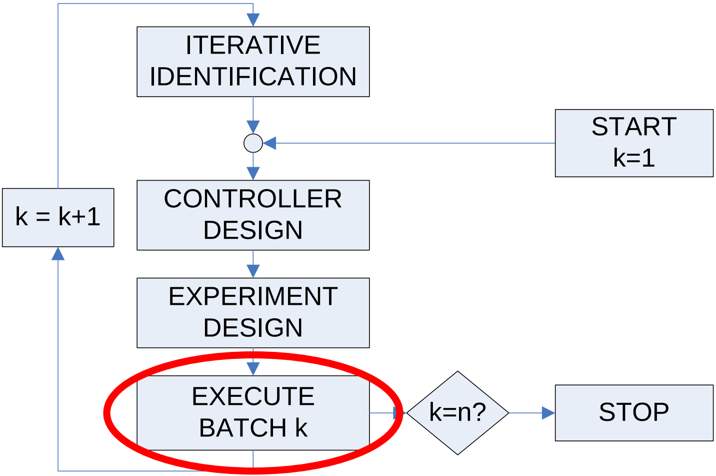
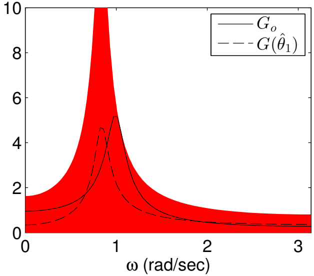
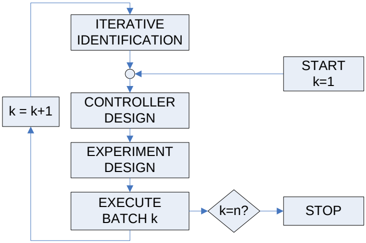


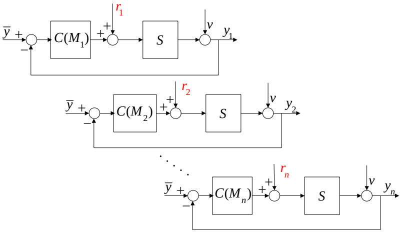
Numerical Example

Second-order system S_o in a full BJ model structure.

- $N = 2400$ total samples.
- $n = 12$ batches of length 200.
- Constraints:
  - $\mathcal{T}_k \leq 0.7$ for $k = 1, \ldots, 6$.
  - $\mathcal{T}_k \leq 0.005$ for $k = 7, \ldots, 12$.

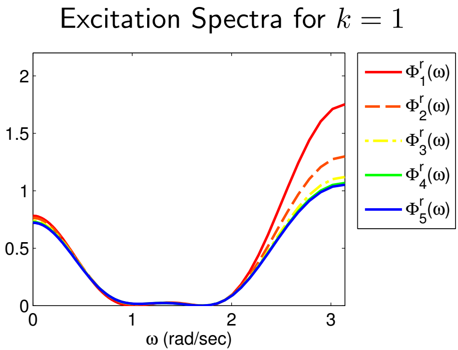


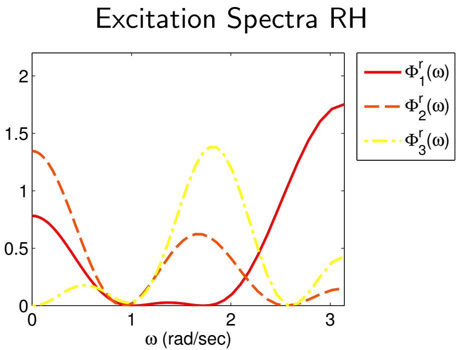




Numerical Simulation

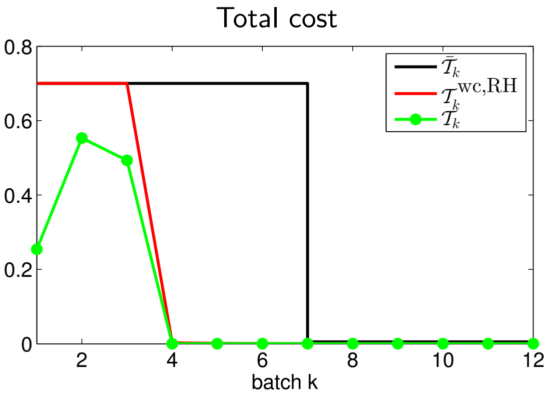
Excitation Spectra

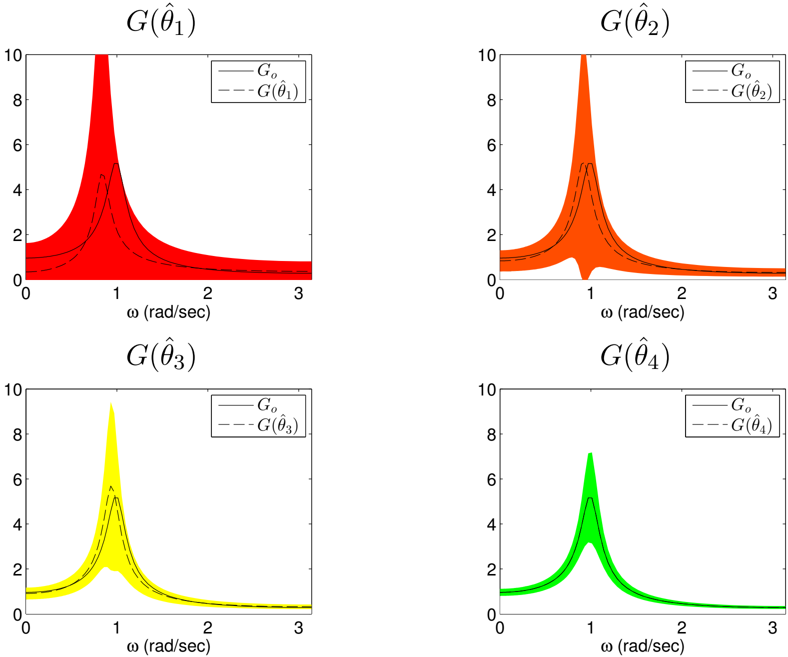




Numerical Simulation

Total cost




$G(\hat{\theta}_1)$
G_o
$G(\hat{\theta}_1)$
ω (rad/sec)
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G_o
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Numerical Simulation

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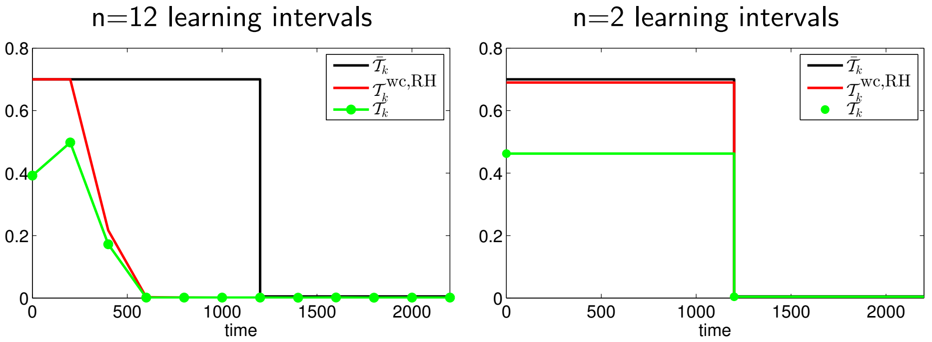

Note: $n = 2$ learning intervals corresponds to a more classical two-phase experiment design.

Intuitively, better performance with shorter intervals. However, asymptotic assumptions in the interval length are used here...

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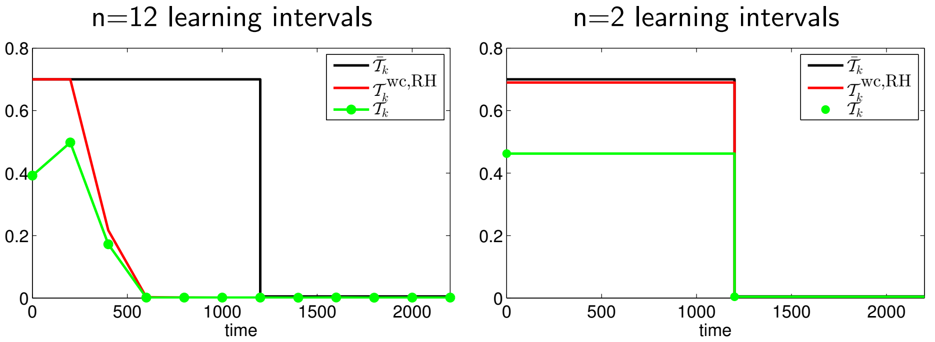

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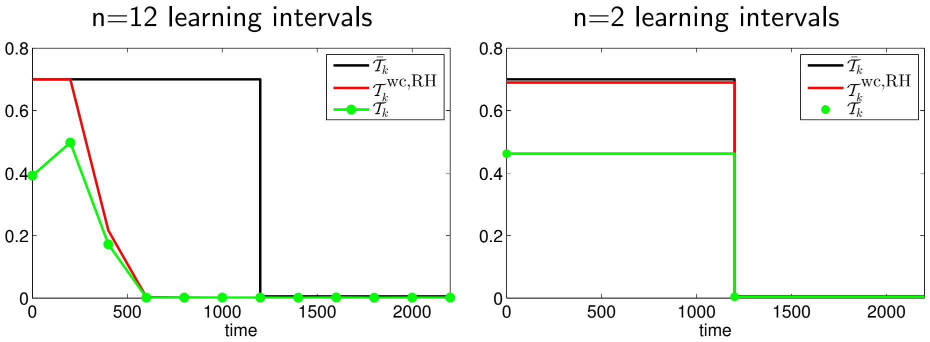

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Conclusions

A framework for iterative model improvement for model-based control.

- Aims to maximize the overall performance.
- No distinction between identification and control batches.
- Excitation is introduced only when it pays back.

The framework was thought for industrial batch processes (learning interval = batch). However, batch processes are often severley nonlinear.

On-going work for nonlinear experiment design.

Thank you.
Questions?

Experiment Design

Worst-case modeling error cost

- From Parseval relation $\mathcal{V}_k = E[(y_k^{ol,e} - y_k^{el,e})^2] =$

$$\mathcal{V}_k(\theta_o, \hat{\theta}_k) = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|\frac{1}{1+C(\hat{\theta}_k)G(\theta_o)} - \frac{1}{1+C(\theta_o)G(\theta_o)}\right|^2 |H|^2(\theta_o)\sigma_e^2\, d\omega$$

- We approximate $\mathcal{V}_k(\theta_o, \hat{\theta}_k)$ as a quadratic function of θ_o locally around $\hat{\theta}_k$

$$\mathcal{V}_k(\theta_k + \delta_k, \hat{\theta}_k) \approx \frac{1}{2}\Delta_k^\top V''(\hat{\theta}_k)\delta_k.$$

- Since $\Delta_k = \theta_o - \hat{\theta}_k \sim \mathcal{N}(0, P_k)$, $\Delta_k \in \mathcal{D}_k(\alpha, P_k^{-1})$ w.p. α

$$\mathcal{D}_k(\alpha, P_k^{-1}) = \{\delta \in \mathbb{R}^p \mid \delta^\top P_k^{-1}\delta \leq \chi_p^2(\alpha)\}$$

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Worst-case modeling error cost

The optimization problem becomes

$$\mathcal{V}^{\text{wc}} = \max_{\delta} \frac{1}{2}\delta^\top V''\delta \qquad \text{such that} \qquad \delta^\top P^{-1}\delta \leq \chi_p^2(\alpha)$$

Using the S-procedure, it is equivalent to...

$$\mathcal{V}^{\text{wc}} = \min_{\lambda} \frac{1}{\lambda} \qquad \text{such that} \qquad P^{-1} \geq \frac{\lambda V''\chi_p^2(\alpha)}{2}$$

that is convex (in P^{-1}).

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Experiment Design

Worst-case excitation cost

The excitation cost

$$\mathcal{E}_k(\theta_o, \hat{\theta}_k) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left| \frac{G(\theta_o)}{1 + C(\hat{\theta}_k) G(\theta_o)} \right|^2 \Phi_k^r(\omega)\, d\omega = {R_k}^\top c(\hat{\theta}_o, \theta_k).$$

where R_k are the coefficients parametrizing $\Phi_k^r(\omega)$

$$\mathcal{E}(\hat{\theta}_k + \delta_k, \theta_k) \approx R_k^\top c(\hat{\theta}_k, \hat{\theta}_k) + R_k^\top J_c(\hat{\theta}_k)\delta_k + \frac{1}{2}\delta_k^\top \left(\sum C_j(\hat{\theta}_k) R_k(j) \right) \delta_k$$

Second order expansion in δ_k, linearly dependent in R_k.

Experiment Design

Worst-case excitation cost

The optimization problem becomes

$$\mathcal{E}^{\mathrm{wc}} = \max_{\delta} \; c_0 + R^\top J_c \delta + \frac{1}{2}\delta^\top \left(\sum C_j R(j)\right)\delta \qquad \text{such that}$$

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Bilinear if P^{-1} depends on variables. P_k^{-1} kept constant to P_1^{-1}...

Experiment Design

Worst-case excitation cost

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Using the S-procedure, it is equivalent to. . .

$$\mathcal{E}^{\rm wc} = \min_{\gamma,\tau}\gamma \ \text{ such that}$$
$$\tau \geq 0$$
$$\begin{bmatrix} \frac{1}{2}\sum_j C_j R(j) - \tau\frac{P^{-1}}{\chi^2} & \frac{1}{2}(R^\top J_c)^\top \\ \frac{1}{2}R_k^\top J_c & R^\top c_o + \tau_k - \gamma_k \end{bmatrix} \leq 0$$

Bilinear if P^{-1} depends on variables. P_k^{-1} kept constant to P_1^{-1}. . .

Experiment Design

Worst-case excitation cost

The optimization problem becomes

$$\mathcal{E}^{\mathrm{wc}} = \max_{\delta} \; c_0 + R^\top J_c \delta + \frac{1}{2}\delta^\top \left(\sum C_j R(j)\right)\delta \qquad \text{such that}$$

$$\delta^\top P^{-1}\delta \leq \chi^2_p(\alpha)$$

Using the S-procedure, it is equivalent to. . .

$$\mathcal{E}^{\mathrm{wc}} = \min_{\gamma,\tau} \gamma \;\; \text{such that}$$

$$\tau \geq 0$$

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Bilinear if P^{-1} depends on variables. P_k^{-1} kept constant to P_1^{-1}. . .

Worst-case excitation cost

Sample-based approach

A sample-based approach...

Using the distribution of $\Delta_1 = \theta_o - \hat{\theta}_1 \sim \mathcal{N}(0, P_1)$:

1. Draw q samples $\tilde{\delta}_s$.
2. Compute $\mathcal{E}_{k,s} = \mathcal{E}_k(\hat{\theta}_k + \tilde{\delta}_s, \theta_k)$ for $s = 1, \ldots, q$.
3. Extract the empirical maximum $\mathcal{E}_k^{\mathrm{wc}} = \max_s \ \mathcal{E}_{k,s}$.

The number of samples q can be tuned such that $\mathcal{E}_k^{\mathrm{wc}}$ is the Worst Case Excitation Cost with probability α (randomized algorithms).

Sample extracted from Δ_1. Similar approximation as taking P_k^{-1} to P_1^{-1} for the second order approach.

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